Chaos: Feature of the Model or Property of Nature?

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Abstract

In this paper, a number of important intertwined questions will be explored. The central question is, what chaos tells us about whether or not the world behaves deterministically. To begin with, chaos must be defined. Next, the relation between models of the world and the world itself must be understood. This involves determining what a model is, and how it can relate and refer to the world. A description of how we attribute chaos to the world, or to the model, is given. Also, the relation between quantum mechanics and chaotic phenomena is explored. In effect, ellements of chaos theory can form a bridge between the quantum realm and macroscopic systems. One can therefore use chaos theory to translate the question of determinism on any level of reality to one about the correct interpretation of quantum theory.

1 Introduction

Chaos theory simply studies a certain class of dynamical systems which arise quite naturally within mathematics and its applications. A property of these systems is that they are in fact deterministic, that is: the evolution of the system is the same iff the initial conditions are the same. But then we have quantum theory stating that on a fundamental level, initial conditions cannot determined with arbitrary precision. Here we have tragedies blows at horizon: it seems that the physical universe itself probabilistic, i.e. essentially undetermined. This raises the question whether or not there is in essence a chaotic aspect of Nature.

In this essay we will concern ourselves with some philosophical aspects of chaos. The central questions are: Is chaos an essential and intrinsic feature of Nature, or is chaos merely a part of our models? That is: what can we say about the ontological status of chaos in the physical world? In order for us to give sensible answers we need to come to terms about the concept of chaos, i.e. what do we call chaos or chaotic behavior. Although the concept of chaos is intuitively quite easily grasped, giving a formal definition turns out to be somewhat problematic. Chaos arises when we make a mathematical model of systems in the physical world. It seems clear that this chaotic behavior is a part of the model, but is it also an essential part of the system to which the model refers? In order to answer this question we need to understand and be clear about what models are, how they refer to the the real world and to which part or subsystems of Nature they are referring. Then we will take a closer look at how we attribute chaos through these models to physical reality. Chaos theory could also function as a bridge between the classical and the quantum scales of reality.

2 What is Chaos?

What makes a chaotical dynamical system different from a non-chaotical one? Qualitatively, chaotic behaviour is thought to occur in systems which are unusually sensitive to initial conditions, and exhibit unstable and aperiodic behaviour. This can only be a property of non-linear equations. So all chaotic models contain non-linear equations. The properties of sensitivity to initial conditions and of unstable and aperiodic dynamics can be properly defined using rspectively

There are various definitions of chaos available and they each focus on more or less their own type of dynamical system. For an example of the variety of definitions for chaos in dynamical systems see [4], wherein the following types of definitions for chaos are listed: Li-Yorke chaos, Experimentalists' definition, Devaney's definition, Wiggins definition and Martelli's definition. In this section we start out by looking at some qualitative aspects of chaos and then we turn to see how chaos in dynamical systems is defined. By doing this we hope to take some steps towards answering the question whether chaos is actually an aspect of physical reality or just part of our models.

The most general qualitative definition of chaos is "unstable aperiodic behaviour in deterministic nonlinear dynamical systems" [1]. This definition restricts chaos to be only an aspect of certain dynamical systems. It is this kind of behaviour that is characteristic of chaos, and by arguments about models in general we suspect that it is actually an aspect of certain physical systems.

Another qualitative aspect of chaos is that it renders long-term prediction of the system impossible, i.e. the system appears to behave in a random fashion. In [3] two attempts of defining chaos through this randomness aspect are discussed. Note that if a system would exhibit only predictable behaviour then we would not characterize it as chaotic, and therefore apperant randomness or unpredictability is a necessary condition for chaotical systems, but according to Batterman it is not sufficient. It is argued that for continuously evolving Hamiltonian systems (system where the energy is reserved), e.g. throwing a die or a table of roulettes, we have complete predictability in the model, and thus randomness is not really an aspect of the system. Others claim that the randomness aspect is brought in via the initial conditions of the system. If the initial conditions consist of non-computable (algorithmically random) numbers then the dynamical system displays the randomness from it's initial conditions. This claim does not hold very well, since, if we restrict the tent map to the computable reals in a bounded interval we still find chaotic behaviour [**?**].

In the next section we will discuss the problem of defining chaos in a formal way.

2.1 Terminology

In order for us to be (as) clear (as we can) on what we mean by chaos in dynamical systems, we should treat dynamical systems first. We present some definitions and some nice theorems about dynamical systems. A dynamical system can be thought of as a description of the evolution of points in some space, often called the *state space*. Here we discuss two types of dynamical systems: smooth and discrete dynamical systems.

Definition 2.1 (Dynamical System)**.** *A dynamical system is a set S called the statespace, a monoid $(I, +, 0)$ called evolution paramater space and a function, $\phi : U \subset I \times S \to S$, which satisfies:*

1. *$\phi(0, x) = x$ the identity function on M*

2. $\phi(t, \phi(s, x)) = \phi(t + s, x)$ *for every* $t, s \in I$ *such that* $t + s \in I$.

In the context of chaos this definition is a little too abstract; we have for instance no direction of time, but it is the most general form of the concept of dynamical system that we could think of. We give the definition for a smooth dynamical system (from: [2]):

Definition 2.2 (Smooth Dynamical System)**.** *A smooth dynamical system on* $\mathbb{R}^n$ *is a continiously differentiable function* $\phi : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$*, where* $\phi(t, x) = \phi_t(x)$ *satisfies:*

1. ϕ_0 *is the identity function on* $\mathbb{R}^n$

2. *The composition* $\phi_t \circ \phi_s = \phi_{t+s}$ *for each* $t, s \in \mathbb{R}$

Definition 2.3. *Discrete Dynamical System A discrete dynamical system on* $\mathbb{R}^n$ *is a function* $\phi : \mathbb{Z} \times \mathbb{R}^n \to \mathbb{R}^n$*, where* $\phi(t, x) = \phi_t(x)$ *satisfies:*

1. ϕ_0 *is the identity function on* $\mathbb{R}^n$

2. *The composition* $\phi_t \circ \phi_s = \phi_{t+s}$*, for each* $t, s \in \mathbb{Z}$.

Note that from the definition of discrete dynamical system it follows by induction that it is of the form $\phi(n, x) = \phi^n(x)$, the n^{th} iterate of ϕ on x. Often one writes $\phi_t(x)$ instead of $\phi(t, x)$. Since we are interested in chaos, we should of course be looking at nonlinear dynamical systems. A dynamical system is *linear* if the function is linear in the statespace, and a dynamical system is nonlinear if it is not linear. We can now define what we mean by a *deterministic* dynamical system.

Definition 2.4 (Deterministic Dynamical System)**.** *A dynamical system is* deterministic *if every state is always followed by the same trajectory.*

Using the fact that smooth and discrete dynamical systems use *functions* the determine the evolutions of a point we conclude that both smooth and discrete dynamical systems are deterministic.

Another important notion is the notion of Orbit of a point in a dynamical system:

Definition 2.5 (Orbit)**.** *The* orbit $O(x)$ *of a point* x *in the statespace of a dynamical system* $\phi_t(x)$ *is defined as the set:* $O(x) := \{\phi(t, x) \mid t \in I\}$*, where* I *is the evolution parameter space.*

The orbit of a point in the statespace of a system is the time evolution of that point within the system. An orbit $O(x)$ is *periodic* if there is some T such that $\phi_{t+T}(x) = \phi_t(x)$, for every t. The *period* is the smallest possible T such that the previous equation holds. For discrete dynamical systems the *forward orbit* of a point x is defined as $\{\phi^n(x) \mid n = 0, 1, 2, \ldots\}$, and in smooth dynamical systems it is defined as $\{\phi(t, x) \mid t \geq 0\}$.

Note that for smooth and discrete dynamical systems the evolution or the orbit of a point of the system is completely determined by the initial values of the system. We claim that if we have that $\phi(t, x) = \phi(t', x')$, then we have $O(x) = O(x')$. Which is demonstrated as follows: Let $y \in O(x)$, or in other words $y = \phi(t_1, x)$ for some t_1. Using the assumption and writing $t = t_1 - t'_1$ we have $y = \phi(t_1, x) = \phi(t + t'_1, x) = \phi(t'_1, \phi(t, x)) = \phi(t'_1, \phi(t', x')) \in O(x')$, and hence $O(x) \subset O(x')$. Interchanging x with x' and t with t' yields $O(x') \subset O(x)$. In other words intersecting orbits are equal, or the orbits form a partition of the statespace.

2.2 Defining Chaos

Most definitions of Chaos in a dynamical system include the notions of sensitive dependance (on initial conditions), (a)periodic orbits and/or aperiodicity and topological transitivity. We will explain these notions and then discuss some popular defintions.

Sensitive Dependance (SD) in a system is characterized by that a small difference in initial conditions of a system yields a vast difference in evolution of the system. There are mainly two types of Sensitive Dependance, the Weak Sensitivity Dependance and the Strong Sensitivity Dependance.

They can be formally defined as [1]:

Definition 2.6 (Weak Sensitivity Dependance)**.** *A dynamical system $\phi_t(x)$ has* WSD *if there is an $\epsilon > 0$ such that for every $\delta > 0$ and every x_0 there exists a y_0 and a $t > 0$ such that: $|x_0 - y_0| < \delta$ and $|\phi_t(x_0) - \phi_t(y_0)| > \epsilon$.*

This definition tells us that for every two different but arbitrary close initial conditions, the evolutions of the system will, at some time t, differ more than some non-zero value.

Definition 2.7 (Strong Sensitivity Dependance)**.** *A dynamical system $\phi_t(x)$ has* SSD *if there is a λ such that almost all x_0 we have: for all $\delta > 0$ there is a $t > 0$ such that for all y_0 in a small neighbourhood of x_0 we have $|\phi_t(x_0) - \phi_t(y_0)| \approx |x_0 - y_0| e^{\lambda t}$.*

The λ in this definition is called the *Lyapunov exponent.* If the Lyapunov exponent is greater than zero, then this definition tells us that for almost every two different but arbitrary close initial conditions, the evolutions of the system will diverge exponentially. The "for almost all" means "except for a subset with measure zero". Both of these definitions can, in a straightforward way be translated to the case of discrete dynamical systems.

Another characteristic feature of Chaotic Dynamical systems is topological transitivity.

Definition 2.8 (Topological Transitivity/Mixing)**.** *A continous surjective map $f : M \to M$ is* topologically transitive *if for every two open sets $U, V \subset M$ there is a $n \in \mathbb{N}$ such that $f^n(U) \cap V \neq \emptyset$. If there is an $N \in \mathbb{N}$ such that for all $n > N$ we have $f^n(U) \cap V \neq \emptyset$, then f is called* topologically mixing*.*

Now we are equipped with enough terminology to discuss a definition of chaos. The following definition applies only to discrete dynamical systems.

A famous definition of Chaos is Devaney's Definition of Chaos:

Definition 2.9 (Devaney's Definition of Chaos)**.** *A map $F : X \to X$ is Chaotic if the following hold:*

1. *F is topologically transitive in X*
2. *F has sensitive dependance in X*
3. *The set P of periodic orbits of F is dense in X.*

In [6] this definition is shown to be redundant; the following theorem is presented. Of course the proof is due to [6], but we resate it and fill in some of the details.

Theorem 2.10. *Let (X, d) be a metric space and $f : X \to X$ a continous bijection. If f is topologically transitive and the set of periodic points is dense in X then f has (weak) sensitive dependence on initial conditions.*

Proof. Assume that $f : X \to X$ is topologically transitive and has sensitive dependence on initial conditions on X.

First we claim that there is a $\delta_0 > 0$ such that for any $x \in X$ we can find a periodic point $q \in X$ such that x has at least distance $\frac{\delta_0}{2}$ to $O(q)$. This can be seen as follows: Let p, q be periodic points in X such that the orbits $O(p), O(q)$ are disjoint. Let δ_0 denote the minimum distance between

$O(p)$ and $O(q)$. By the triangle inequality it follows that for every $x \in X$, $p' \in O(p)$ and $q' \in O(q)$ we have

$$\delta_0 \leq d(p', q') \leq d(p', x) + d(x, q').$$

Taking the minimum over $p' \in O(p)$, $q' \in O(q)$ yields

$$\delta_0 \leq \min_{p'} d(p', x) + \min_{q'} d(x, q').$$

Hence every $x \in X$ has at least distance $\frac{\delta_0}{2}$ to one of the orbits $O(p)$, $O(q)$. This demonstrates the claim, and we now return to the proof of the main statement: we will show that f has sensitive dependence with sensitivity constant $\delta = \frac{\delta_0}{8}$

Let x be an arbitrary point in X and let U_x be some neighbourhood of x. Consider the open set $U = U_x \cap B_\delta(x)$. Since the periodic points of f are dense in X, there is a periodic point $p \in U$ of f with period n. By the previous claim, we can find a periodic $q \in X$ such that the distance between x and $O(q)$ is at least 4δ. Consider the following set:

$$V = \bigcap_{i=0}^{n} f^{-i}(B_\delta(f^i(q)).$$

First note that V is non-empty; we have that $q \in f^{-i}(B_\delta(f^i(q)))$ for every $i = 0, 1, \ldots, n$. Since f is continous we have that all the $f^{-i}(B_\delta(f^i(q)))$ are open and since V is a finite intersection, V is itself open. Recall that n is the period of $p \in U$. Because f is topologically transitive, there exists a $y \in U$ and a $k \in \mathbb{N}$ such that $f^k(y) \in V$. Let j denote the integer part of $k/n + 1$; since $n \geq 1$ and $j \geq 1$ we have $1 \leq nj - k \leq n$. By construction of V we have:

$$f^{nj}(y) = f^{nj-k}(f^k(y)) \in f^{nj-k}(V) \subset B_\delta(f^{nj-k}(q)).$$

Also we have that $f^{nj}(p) = p$. Subsequent application of the reverse triangle inequality yields:

$$\begin{aligned} d(f^{nj}(p), f^{nj}(y)) &= d(p, f^{nj}(y)) \\ &\geq d(x, f^{nj}(y)) - d(x, p) \\ &\geq d(x, f^{nj-k}(q)) - d(f^{nj-k}(q), f^{nj}(y)) - d(x, p) \\ &> 4\delta - \delta - \delta = 2\delta. \end{aligned}$$

The last strict inequality follows from the following: q was chosen such that x has a distance of at least 4δ to $O(q)$, and thus $d(x, f^{nj-k}(q)) \geq 4\delta$, morover since $p \in B_\delta(x)$ and $f^{nj}(y) \in B_\delta(f^{nj-k}(q))$ we also have $d(p, x) < \delta$ and $d(f^{nj-k}(q), f^{nj}(y) < \delta$. Using the triangle inequality yet once more, we find:

$$d(f^{nj}(p), f^{nj}(x)) + d(f^{nj}(x), f^{nj}(y)) > 2\delta$$

and thus either $d(f^{nj}(p), f^{nj}(x)) > \delta$ or $d(f^{nj}(x), f^{nj}(y)) > \delta$ is the case. In any case we have found a point in U_x whose nj^{th} iterate has distance greater than δ from $f^{nj}(x)$. In other words: f has sensitivity of initial conditions on X.

□

If the sensitive dependence on initial conditions requirement is removed from the definition, it becomes less intuitive that this definition has to do with chaos. A reformulation of the two essential properties can be given, as seen in [5]. There it is said that, under the same conditions as in Devaney's definition, a map f is chaotic if for every two non-empty open $U, V \subset X$ there is a periodic point $p \in U$ and a $k \in \mathbb{N}$ such that $f^k(p) \in V$. Then it is shown that f is chaotic in this sense if and only if f is topologically transitive and that it's periodic points are dense in X.

Now we have seen that it is quite possible to give a definition of chaos in the context of deterministic dynamical systems. Although the definitions presented here only apply to discrete dynamical systems, we are confident that a definition of chaos for smooth dynamical systems can be given. Even though we were unable to find such a defintion or to give one, we assume that it can be done. By defining chaos we should have convinced the reader that it exists in the formal framework of dynamical systems, and therefore that it is truly a part of some of our models. The question of whether or not chaos exists in the physical world is now shifted to the question of how one attributes the features of a model to the physical world.

3 What is a Model? [7]

Models are supposed to be about things in the world. In order to do this, they must be able to refer to the world. How is this done? And what is the relation between a model ande a system? These are the questions that wil be explored next.

There are basically two general ideas about what scientific theories are [8]. In the syntactic view, a theory is a set of sentences in an axiomatic structures in a first-order logic. In this view, a model is just another way of saying the same thing a theory can say, models are therefore of no fundamental importance in science. According to the semantical view of theories and models, a theory is basically a collection of models. The axiomatization of what these models have to say, can be dispensed with without losing the relevant science. A model can be regarded as either a linguistical thing, or as non-linguistic entities. The linguistic approach sees models as descriptions that have some form of reference to the outside world. This inherrits all the philosophical problems of the Philosophy of Language. The alternative is faced with the question what a reference of a model is, if a model is not a linguistical entity. Models can exist in various forms. They can be seen as physical objects or fictional objects. More ways to think of models to exist are as set-theoretic structures, descriptions, equations or as a combination of these.

When this is applied to models in chaos theory, we have to look at how the underlying theories are represented in specific models that describe systems. It seems the case that most theories, seen as axiamatized structures, do not have the property inpredictability. The models we build with our theories do have this property, if combined with the fact that we do not have infinitely precise knowledge of the initial conditions. If this is true, then the realist will have to choose between realism towards the theory, or realism towards the model. If the first option is followed, it can be maintained that chaos is a property of the model alone, and not of the world. If, on the other hand, models are considered to represent real entities (instead of axiomatized theories), then chaos is a verry real phenomenon and the theories alone cannot be representative for the state of affairs out there.

From a physicalistical point of view, models are both invaluable devices, and problematic entities. Scientific models tell us what the world is like, without containing teleological or spiritual notions. They therefore can form some kind of justification for the physicalist, which is indispencible. In a different way, however, they are troublesome. Since models must be concidered as existing objects, they themselves are part of the (physical) reality. However, many philosophers wondered how something purely physical can refer to other physical things. It is said that models refer by analogy or some kind of isomorphism, it seems that these concepts are extremely vague. The problem of reference is particularly accute here.

4 Chaos, Quantum Mechanics and Determinism

It seems possible that chaos in models is, on first account, not inconsistent with determinism. After all, if the Laplacian demon can classically know all about the world with infinite precission, he wouldn't even notice whether there is chaos in the world or not. However, this way of thinking about matters might have to be abandened in the light of some properties of quantum mechanics. According to the Heisenberg uncertainty principle, information belonging to non-commuting operators cannot simultaniously be known to an arbitrary degree. This would mean that a state with all logically possible information specified, is physically impossible. It must be remembered, though, that this version of a quantum property of the world relies on the Copenhagen interpretation of quantum theory. There is still a lively debate in the philosophy of science about what quantum physics really tells us [**?**]. Ian Stewart explaines why someone like Einstein, who was a determinist, would have liked the idea of understanding quantum mechanical phenomena as examples of classical chaos. After all, there can still be determinism in this case, and there is no need for a probabilistical interpretation in the vein of pressent-day quantum mechanics. According to Stewart, the best place to start looking for a revission in Q.M. is the lack of a clear theoretical description of what happens when a measurement is made in a quantum system. In a measurement, there is always one outcome. The quantum state, however, is a superposition. Therefore measurement of a quantum system is only possible with a macroscopic aparatus, this is precisely the Copenhagen interpretation with the "collapse of the wavefunction". One might remark, however, that the difference in size of the quantum system with respect to the measuring device is only a relative difference, not absolute. Quantum mechanics should be about all systems, not just about small things. Furthermore, the Copenhagen interpretation of Quantum mechanics neglects the dynamical evolution of the wavefunction when system is measured; the "collapse of the wavefunction" is an instantaniously occuring phenomenon. Stewert also discusses the Einstein-Podoski-Rosen paradox. This entails the difficulty of non-locality when "entangled" particles are concidderred. A possible solution for the problems, and therefore an alternative of, the Copenhagen interpretation is the mathematical formalism of David Bohm. Instead of introducing the concept of the collapse of the wavefunction, Bohm argued that all particles in the universe obey the Schrödinger equation. In addition, he introduced equations that describe our ignorence of the state of the system, and those that describe the movement of the particle as a function of the wavefunction. The combination of these properties makes Bohm's framework totally deterministic. Stewart argues that Bohm's work has been neglected by the physical community without good reasons. Agaist the idea of hidden variables, which would explain our incomplete knowledge of a state in a deterministical fashion, the Bell's inequallity was conceived. The Bell's inequality can be constructed using a thought experiment in the following way, following Stewart's book. When two spin $\frac{1}{2}$ particles are produced in an entangled state, the spin orintation of each particle can be measured in one direction. Under the assumption of locality and determinism, a relation that describes the correspondence between erach of the spin measurements and the angle in which the spin of one electron is measured relative to the measurement of the other spin measurement, can be derived. This is known as Bell's inequality [**?**]:

$$1 + C(b, c) \geq |C(a, b) - C(a, c)|,$$

in which C is the correlation of the spin measurements of two sets of electrons at different relative angles a, b, and c. This inequality is inconsistent with quantum mechanical predictions, and inconsistent with experiments. It is to be concluded that eiter determinism, or locality, or both, fail.

Independent of our interpretation of quantummechanics, however, chaos is manifest in many systems in the semi-classical and the non-classical domain ([9], chapter 10). The semiclassical domain is a crossover region between the classical and the non-classical realms. A question of particular importance is: what implications does chaotical classical dynamics have for the quantum description of a system in the semiclassical regime? This question can be eluminated using a classical example which resembles the schrödinger equation. The Helmolz wave equation resembles the Schrödinger equation, as described in [9]: $\nabla^2\Psi + k^2\Psi = 0$, where k is the wavenumber. If the correspondence is worked out in detail, the conclusion is that chaotic features of the classical case can help to say something of the semiclassical, and even of the quantum mechanical case.

The relation between quantum mechanics and chaos is intriguing. Some argue that since quantummechanical phenomena only manifest themselves in the realm of the microscopic, an interpretation of quantum mechanics cannot influence the idea of (in)determinism in macroscopic systems. However, it is confirmed that infinitesimal quantum effects on the subatiomic level can eventually influence macroscopical systems. This idea, closely linked with the stability number of mappings, might argue for a genuine indeterministic account of chaotic processes. Thus chaos turns out to form a bridge from the microscopic to the macroscopic. Finding out the correct interpretation of quantum mechanics then becomes essential.

5 Wittgenstein and the Tractatus Logico-Philosophicus

It is necessary to clarify what we mean exactly by determinism. It seems natural that you can see determinism as either a property of our models, or as a property of the natural world (or both). In 1922, Wittgenstein published his influential work on language and the world. His paper is an example of a syntactic view on science. It is not neccessary to treat his early philosophy here, in order to clarify an important aspect of the notion of determinism. Wittgenstein looks upon the logically deducability of propositions as uniquely dependent upon the truth functions of these propositions. It is redundent to postulate "rules of inference", which would justify this deducing, because the meaning of mutual deducible propositions is such that the deducability is internal to the propositions. When this is applied to two propositions that are said to represent two different states of affairs in the world, it seems obvious that they have a different truth-value. Therefore, unlike the case of two logically derivable propositions, the meaning of the two descriptions of states does not entail any connection between them. In particular, any causal chain which might be said to exist between the propositions, is not internal to the propositions, but alwys something external to it (something we add to it). Therefore, Wittgenstein concludes, to say that one state of affairs causes another is "superstition" [10, 5.132-5.161]. This shows, that if you agree with Wittgenstein, any notion of determinism which relies on "linguistical" causality (the causal relation between the meanings of sentences) must be abandoned. It therefore cannot be maintained that determinism in dynamical systems is a logical relation between propositions that express different states of affairs. To speak of another kind of determinism would deffenitely not be advised by Wittgenstein, because all we can talk about are linguistical items.

Still, his view leaves open the possibility of determinism "in nature", though such a view would be a metaphysical statement. We can see, however, that Wittgenstein himself was some kind of a determinist, in so far as he thought that the human mind does not have the capacity to change the physical reality [10].

6 Conclusion

We have seen that chaos can be formally defined within the context of dynamical systems.

Smooth and discrete dynamical systems are deterministic in the sense that same points in the statespace are always followed by the same statespace trajectory. We have seen that via the (early) philosophy of Ludwig Wittgenstein determinism should not be thought of as if it would entail a logical relation between propositions.

How to interpret chaos as a property of models is without significant problems, how to interpret it as a property of physical reality remains problematic although it can be clarified if viewed in the light of the discussion of a semantic versus syntactic interpretation of science. The authors uphold the syntactic view on science. However insofar the models are logically derived from the postulates and the specific boundary conditions, they bear significance as scientific objects. Models are then considered to be exstensions of the theory and their properties are not considered to be primary but their propositional content displays information about the world. In good science certain sets of initial conditions are consistent with the postulates of a theory and it is to be understood that models are obtained by filling in the boundary conditions of the target system.

The unpredictability of chaotic dynamical systems is not a logical consequence of the postulates of the theory, unless impossibility of perfect knowledge of states of affairs is a logical consequence of the postulates.

Since there exist various examples of chaotic dynamical systems in science, we are forced to conclude that our best postulates entail chaos under certain boundary conditions.

Furthermore we have been able to reduce the question of determinism to a question of the correct interpretation of quantum physics. As far as the authors are concerned the question whether the indeterministic nature of quantum phenomena is a real aspect of nature is not yet settled. We have seen that the Copenhagen interpretation is problematic in the description of the measurement. We conclude that an open mind should be maintained towards other interpretations of quantum mechanics, and therefore towards determinism on any scale of reality.

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